



# Numerical Models for Simulating Ocean Physics

Michael J. Bell<sup>1</sup>, Yann Drillet<sup>2</sup>, Matthew Martin<sup>1</sup>, Andreas Schiller<sup>3</sup>, Stefania Ciliberti<sup>4</sup> <sup>1</sup>MetOffice, Exeter, UK <sup>2</sup>Mercator Ocean International, Toulouse, France <sup>3</sup>CSIRO Environment, Castray Esplanade, Hobart, Tasmania, Australia

<sup>3</sup>CSIRO Environment, Castray Esplanade, Hobart, Tasmania, Australia
 <sup>4</sup>Nologin Oceanic Weather Systems, Santiago de Compostela, Spain

Correspondence to: Mike Bell (mike.bell@metoffice.gov.uk)

10 Abstract. We describe, at an elementary level, the spatially varying properties of the ocean that physical ocean models represent, the principles they use to evolve these properties with time, the physical phenomena that they simulate, and some of the roles these phenomena play within the Earth system. We also describe, in some technical detail, the methods and approximations that the models use and the difficulties that limit their accuracy or reliability.

#### **1** Introduction

- 15 The models of ocean physics described in this paper, use physical principles to simulate how the three-dimensional structures of the ocean's temperature, salinity and currents evolve in time. Section 2 describes the models at an introductory level. It outlines first the spatially varying quantities they predict and the physical principles they use. It then describes the circulations the models simulate and some of the reasons why these circulations are important in the Earth system. Section 3 outlines at a more technical level the main approximations the models typically use and the steps in the discretisation of their equations,
- 20 drawing attention to some of the difficulties which limit their accuracy or reliability. Section 4 of Wan et al. (2024) describes aspects of the design, testing, documentation and support for an ocean model code that are crucial for it to be suitable for use in operational predictions or climate simulations.

#### 2. An overview of the models and what they simulate

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#### 2.1 The quantities simulated and the principles used

The temperature structure of the ocean at a given time in a physical ocean model is represented by a three-dimensional (3D) grid of temperature values. The three dimensions of the grid correspond to the three dimensions of space. One of the dimensions

30 is aligned with the local vertical and the other two with local horizontal directions. The set of temperature values on the grid is referred to as the temperature field. The salinity structure is similarly represented by a 3D grid of salinity values, referred to as the salinity field. The horizontal currents are represented by two fields, representing the currents in two locally horizontal





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directions and the locally vertical current by a third field. The fluid's density and pressure are also represented by fields. In total, conceptually there are seven 3D fields (the temperature, salinity, density, pressure and 3 velocity fields) and the physical

35 ocean model simulates how these fields will evolve in time. Given all these fields at time t, the model predicts how they will all evolve over the next few minutes or hours, that is over a time-step  $\Delta t$ , and hence their values at time  $t + \Delta t$ . Model predictions to days, months or years ahead are generated by performing a large number of time-steps.

The equations used by physical ocean models are based on the physical principles of:

- conservation of momentum (Newton's laws of motion) for each direction in space;
  - conservation of the mass of water and salt;
  - conservation of energy (the first law of thermodynamics);
  - the thermodynamics determining the density at a point from the temperature, salinity and pressure (the equation of state).
- 45 These 7 sets of constraints are sufficient to determine how the 7 fields will evolve. In practice, the details of how the equations are used to provide computationally efficient, stable and accurate solutions are quite intricate. The accuracy of the model predictions is primarily limited by the representation of the ocean structure by the values on a grid whose resolution is limited by computational power. Motions at scales smaller than the grid are not resolved. The effects of these motions on the resolved scales are calculated by parametrisation schemes. Although these are based on physical principles and detailed studies, their
- 50 accuracy and reliability are inevitably limited. This is one of the main areas where further research has potential to improve the model simulations.

#### 2.2 The circulations simulated and their impacts

- 55 The circulations and physical phenomena that these ocean models are typically used to simulate are principally the:
  - near-surface boundary layer where there is strong turbulent mixing driven by surface winds and heating or cooling;
  - gyre circulations associated with the region, called the thermocline, where the vertical density gradient is strongest.
     Large-scale displacements in the thermocline are primarily driven by Ekman pumping: in the sub-tropical gyres, the thermocline is bowl-shaped; in the sub-polar gyres it is dome-shaped (chapter 20 of Vallis 2017);
- meridional overturning circulations (MOCs) associated with heat loss and convective mixing at high latitudes and wind driven upwelling near the equator and in the Southern Ocean;
  - western boundary currents associated with the MOCs and the gyre circulations;
  - mesoscale circulations (with horizontal scales < 100 km) associated with instabilities of the boundary currents and gyre circulations;
- sub-mesoscale motions (with horizontal scales < 10 km) that are strongest in the near-surface boundary layer.







These circulations and phenomena play important roles in the Earth system. For example: the western boundary currents are responsible for very large meridional transports of heat and geographically varying air-sea fluxes which help to shape atmospheric circulations; interannual variations in the slope of the thermocline along the equator in the Pacific ocean are an essential component of the El Nino / Southern Ocean (ENSO) phenomenon; the advection of heat by large-scale ocean currents

- 70 towards ice shelves has a significant impact on their heat balance and evolution; and biogeochemical cycles are typically sensitive to the vertical advection of nutrients. The ocean models can be configured as a component of a coupled system, with models of other components such as the atmosphere, sea-ice, surface waves or biogeochemistry, or as a stand-alone system with suitable data sets providing surface
- forcing. They can be configured to cover the entire global ocean, or to cover just a limited region with lateral boundary conditions (that are often taken from a model of a larger region). Their initial conditions can be specified by climatologies based on historical measurements or regularly updated by assimilating the latest measurements as in operational forecast systems. The model coupling, domain, resolution and initial conditions should be chosen to suit the purpose of the modelling and are constrained by the computational resources available.

#### 3 Methods and approximations employed in ocean models

80 Fox-Kemper et al. (2019) provide a good recent review of ocean modelling methods and Griffies (2004) is still a helpful primer on the basic techniques.

### 3.1 Variables and equations used

The ocean models used in physical ocean prediction systems evolve 3D fields of the active tracers and the three components of velocity (see section 5.5.1. of Alvarez-Fanjul et al. 2022). They also evolve either a 2D surface pressure (or surface height)

- 85 field or a 3D pressure field. The active tracers used depend on the formulation of the equation of state. When it is EOS80 (Fofonoff and Millard 1983) the active tracers are potential temperature and practical salinity, whilst when it is TEOS10 (IOC et al. 2010) they are conservative temperature and absolute salinity. The evolution of these fields is determined by some form of the so-called primitive equations (Griffies and Adcroft 2008). The approximations that are usually made are generally well-described in section 5.4 of Alvarez-Fanjul et al. (2022). We note however that the centripetal acceleration is not included in
- 90 the equations because they have been transformed so that the elliptical geoid of the Earth's bulge follows a spherical surface (Vallis 2017). It is of course assumed (the turbulent closure hypothesis) that the effect of small-scale motions on large-scale motions can be represented (that is parametrised) in terms of the large-scale motions. None of the Boussinesq, hydrostatic, incompressible and additional Coriolis term approximations is mandatory but maintaining consistent, well-behaved, equations requires care. Some alternative forms of the primitive equations which enjoy good conservation properties are derived in White
- 95 et al. (2005). Compressible equations support rapidly traveling sound waves which (can be artificially slowed but) make competitively efficient solution difficult.







#### 3.2 Numerical discretization

Ocean models normally use a smoothly varying horizontal grid consisting of triangular or quadrilateral elements (section 5.4.2. of Alvarez-Fanjul et al., 2022). Where the grid-lines on the quadrilateral grids intersect, they are usually orthogonal (hence called curvilinear orthogonal). The grids are chosen to have rather uniform resolution (cubed sphere grid, Ronchi et al., 1996) or to be isotropic (same resolution locally in the two directions) with grid-spacing decreasing away from the equator and the poles of the grid placed over land (Madec and Imbard, 1996). Triangular elements have the obvious advantage that they can be chosen to follow coastlines more accurately. With triangular elements, reduced grid-spacing is often employed for selected regions within one smoothly varying grid. With quadrilateral elements, reduced grid-spacing is usually achieved by using

- 105 separate "child" grids that are nested within the "parent" grid with 1-way nesting (the "child" takes boundary values from the "parent" - Staniforth, 1997) or 2-way nesting (the "parent" also takes values from the "child" - Debreu and Blayo, 2008). Finite difference and finite volume methods are usually employed with the quadrilateral grids. Finite volume models evolve their fields by calculating the fluxes across their cell faces. The cell values in such models should be interpreted as cell mean values (rather than values at a point near the centre of the grid), but the difference between the two is not significant for terms
- 110 that are calculated only to second order accuracy. However the pressure gradient calculation in ROMS and NEMO (Shchepetkin and McWilliams, 2003), which calculates the term to higher order accuracy, interprets variables as point values (rather than cell mean values) in its calculations. Models using triangular elements, use either finite element or finite volume techniques (Danilov, 2010) (FESOM has transitioned from finite element to finite volume).
- The main choices for the staggering of variables on orthogonal grids are the B-grid and C-grid (Arakawa 1960). The dispersion properties of internal waves on the C-grid is better (worse) than the B-grid when the grid resolves (does not resolve) the Rossby radius. Stationary chequer-board modes for the pressure field on the B-grid and the velocity field on the C-grid can be associated with undesirable grid-scale "noise". The dispersion properties of internal waves on triangular grids are more problematic though some finite element pairs (Le Roux et al., 1998) perform relatively well. There has been significant recent progress in the development of C-grid-like formulations for triangular grids (and their hexagonal dual grids) with good, mimetic, properties (Ringler et al., 2010; Cotter and Shipton, 2012).
- The choice of vertical "grid" is well known to have far-reaching consequences for ocean models. Ideally the grid will have fine vertical spacing near the surface, so that the mixed layer can be well represented, and the surfaces on which the vertical coordinate take constant values, will follow isopycnals at mid-depths, so that advective velocities and spurious (numerical) time-mean advective diapycnal transports are minimized, and follow the bathymetry at the ocean bottom, so that flow down
- 125 slopes (with its associated vortex stretching) is well represented. Techniques to use coordinates that treat some parts of the motions using Eulerian methods and others using Lagrangian approaches with re-mapping are described in Petersen et al. (2015), Griffies at al. (2020), Hofmeister et al. (2010). Generation of an appropriate vertical grid for ocean models is an active area of research.







Most terms in ocean models are calculated using second-order accurate formulae. The advection of tracers should however be calculated using schemes of higher order accuracy (typically third or fourth order) which also take care to retain the upper and lower bounds of the advected quantities. There is a very extensive literature on this subject (Brasseur and Jacob, 2017) and it is generally agreed that the advecting velocity field should be constrained to be sufficiently smooth (e.g., Ilicak et al., 2012). Specific terms in the equations of motion present different challenges depending on the grid that has been chosen. For terrainfollowing coordinates, calculation of the horizontal pressure gradient to higher order (Shchepetkin and McWilliams, 2003)

- 135 and of the diffusion along isopycnal surfaces (Lemarié et al., 2011) is beneficial, and some smoothing of the bathymetry is necessary. Formulation of the governing equations for the cells that are only partially filled by water is an active area of research (Adcroft, 2013; Debreu et al., 2020). For C-grid models, calculation of the Coriolis term should ensure conservation of energy and some care is needed to avoid unintended transfer of energy to the grid-scale (Ducousso et al., 2017). The strengths and weaknesses of various time-stepping schemes used in ocean models are reviewed in Lemarié et al. (2015).
- 140 Various approaches have been taken to the time-stepping of the external (barotropic) mode (Shchepetkin and McWilliams, 2003; Demange et al., 2019).

#### 3.3 Parameterization of unresolved processes

The parameterization of unresolved processes is of primary importance: Fox-Kemper et al. (2019) provides a useful review. The classic parameterizations of isopycnal diffusion (Redi, 1982; Visbeck et al., 1997), and of the slumping of isotherms by

- 145 baroclinic instabilities (Gent and McWilliams, 1990) work well in climate models with order 1° grid spacing. The latter needs to be developed further for models of higher resolution using ideas such as Bachman (2017) and Mak et al. (2018). It is increasingly clear that sub-mesoscale motions within the ocean surface boundary layer flux heat vertically (Fox-Kemper et al., 2011) and generate filamentary structure. The interaction of these motions with standard parametrisations of turbulence (Umlauf and Burchard, 2005) and Langmuir turbulence (Reichl et al., 2016) is an active area of research as is the
- 150 parameterization of internal dissipation by internal gravity waves generated by tidal displacements over steep bathymetry (de Lavergne et al., 2020). Machine learning methods are being applied to the parametrisation of sub-gridscale motions (Zanna & Bolton 2020, Ross et al. 2023) and are likely to play important roles in future ocean models.

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## 260 Competing interests

The contact author has declared that none of the authors has any competing interests.

## Data and/or code availability

This can also be included at a later stage, so no problem to define it for the first submission.

#### Authors contribution

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